

Algebraic effects in Montague semantics

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October 28, 2020

- 1 Side effects in linguistic semantics
- 2 Algebraic effects and handlers
- 3 Making it Montagovian
- 4 Quantification and dynamism

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Semantics is for...

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- characterizing semantic knowledge...
 - ▶ ...i.e., knowledge of *entailments?* *distributional properties?*
- describing how linguistic structure (i.e., syntax) gives rise to the things being characterized (whatever they are)
- describing how pragmatic stuff (e.g., presupposing something, referring to something, expressing something) should affect the things being characterized





- used models as a vehicle to characterize meanings in terms of entailments



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- described how linguistic structure gives rise to meanings, *compositionally*



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- used models as a vehicle to characterize meanings in terms of entailments
- described how linguistic structure gives rise to meanings, *compositionally*
 - ▶ simply typed λ -calculus
- **no** pragmatic stuff

Montague 1973:

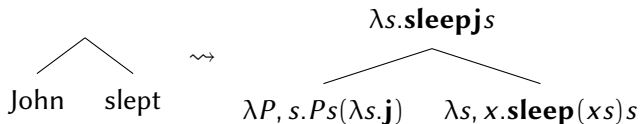
Rules of functional application

- S4. If $\alpha \in P_{t/IV}$ and $\delta \in P_{IV}$, then $F_4(\alpha, \delta) \in P_t$, where $F_4(\alpha, \delta) = \alpha\delta'$ and δ' is the result of replacing the first *verb* (i.e., member of B_{IV} , B_{TV} , $B_{IV/t}$, or $B_{IV//IV}$) in δ by its third person singular present.

Rules of functional application

- T4. If $\delta \in P_{t/IV}$, $\beta \in P_{IV}$, and δ, β translate into δ', β' respectively, then $F_4(\delta, \beta)$ translates into $\delta'(\wedge\beta')$.

- T5. If $\delta \in P_{t/IV}$, $R \in P_{IV}$ and δ, R translate into δ', R' respectively, then $F_4(\delta, R)$



Rules of quantification

- S14. If $\alpha \in P_T$ and $\phi \in P_t$, then $F_{10,n}(\alpha, \phi) \in P_t$, where either (i) α does not have the form \mathbf{he}_k , and $F_{10,n}(\alpha, \phi)$ comes from ϕ by replacing the first occurrence of \mathbf{he}_n or \mathbf{him}_n by α and all other occurrences of \mathbf{he}_n or \mathbf{him}_n by $\left\{ \begin{array}{l} \mathbf{he} \\ \mathbf{she} \\ \mathbf{it} \end{array} \right\}$ or $\left\{ \begin{array}{l} \mathbf{him} \\ \mathbf{her} \\ \mathbf{it} \end{array} \right\}$ respectively, according as the gender of the first B_{CN} or B_T in α is $\left\{ \begin{array}{l} \text{masc.} \\ \text{fem.} \\ \text{neuter} \end{array} \right\}$, or
- (ii) $\alpha = \mathbf{he}_k$, and $F_{10,n}(\alpha, \phi)$ comes from ϕ by replacing all occurrences of \mathbf{he}_n or \mathbf{him}_n by \mathbf{he}_k or \mathbf{him}_k respectively.

Rules of quantification

- T14. If $\alpha \in P_T$, $\phi \in P_t$, and α, ϕ translate into α', ϕ' respectively, then $F_{10,n}(\alpha, \phi)$ translates into $\alpha'(\hat{x}_n \phi')$.

Every dog slept

every dog he_n slept

\rightsquigarrow

$\lambda s. \forall u : \mathbf{dog} \ u s \rightarrow \mathbf{sleep} \ \underline{u} s$

$\lambda P, s. \forall u : \mathbf{dog} \ u s \rightarrow P(\lambda s. u) s$ $\lambda s. \mathbf{sleep}(\underline{x}_n s) s$

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Not compositional

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 - ▶ Idioms (Kobele, 2018)

Programming languages may exhibit "impure" behaviors.

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 - ▶ `def add(x, y):`
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Theories of side effects (e.g., monads) provide interfaces to impure behavior.

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- identity linguistic phenomenon that appears to behave “impurely”, i.e., by subverting compositionality
 - ▶ e.g., quantification, anaphora, conventional implicature...
- find an effectful interface that appropriately describes its behavior
- add it to your compositional repertoire!

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 - ▶ the Continuation monad and the State monad, respectively
 - ▶ analyses inspired by Charlow (2014)
- show how they may and *may not* be combined
- introduce *algebraic effects*

a functor \mathcal{M} , equipped with two operators, $(\cdot)^\eta$ ('return') and $\gg=$ ('bind')

Definition (\mathcal{M})

$$\mathcal{M} : \mathcal{T} \rightarrow \mathcal{T}$$

$$(\cdot)^\eta : a \rightarrow \mathcal{M}(a)$$

$$(\gg=) : \mathcal{M}(a) \rightarrow (a \rightarrow \mathcal{M}(b)) \rightarrow \mathcal{M}(b)$$

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The operators must satisfy the **Monad Laws**.

$$v^n \gg= k = kv \quad \text{(Left Identity)}$$

$$m \gg= \lambda v. v^n = m \quad \text{(Right Identity)}$$

$$(m \gg= n) \gg= o = m \gg= \lambda v. nv \gg= o \quad \text{(Associativity)}$$

First case: quantification

In the Continuation monad, scope-taking is a kind of side effect.

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$$m \gg k = \lambda c. m(\lambda v. kv c)$$

- 1 Ashley hugged every dog.

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ashley = $\mathbf{a}^n : \mathcal{C}(e)$ (Lexicon)

hugged = $\mathbf{hug}^n : \mathcal{C}(e \rightarrow t)$

every = $\lambda P, c. \forall x : Px \rightarrow cx : (e \rightarrow t) \rightarrow (e \rightarrow t) \rightarrow t$

dog = $\mathbf{dog} : e \rightarrow t$

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$$\begin{aligned} m \triangleright n &= m \ggg \lambda f. n \ggg \lambda x. (fx)^\eta \\ &= \lambda c. m(\lambda f. n(\lambda x. c(fx))) \end{aligned}$$

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expand **everydog**...

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$$\mathbf{a}^n \triangleleft (\mathbf{hug}^n \triangleright \lambda c. \forall x : \mathbf{dog} x \rightarrow cx)$$

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expand $\triangleright \dots$

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$$\mathbf{a}^{\eta} \triangleleft \lambda c. \forall x : \mathbf{dog} x \rightarrow c(\mathbf{hug} x)$$

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to obtain a proposition, apply to the identity function...

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$$\forall x : \mathbf{dog}x \rightarrow \mathbf{hug}xa$$

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- scopal expressions take scope over their continuations, which are reified as they compose
- values take scope trivially (applying Montague's "lift")

Second case: anaphora

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(Lexicon)

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$$\text{herself} = \lambda s. \langle s, \text{self} s \rangle : s \rightarrow (s, e)$$

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$(\cdot)^\blacktriangleright : \mathcal{S}(e) \rightarrow \mathcal{S}(e)$

$$m^\blacktriangleright = m \ggg \lambda x, s. \langle x :: s, x \rangle$$

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ashley ▶ ◁ (hugged ▷ herself)

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ashley ▶ ◁ (**hug**ⁿ ▷ herself)

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$$(\lambda s. \langle \mathbf{a}::s, \mathbf{a} \rangle) \triangleleft (\mathbf{hug}^n \triangleright \text{herself})$$

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$$(\lambda s. \langle \mathbf{a}::s, \mathbf{a} \rangle) \triangleleft (\mathbf{hug}^n \triangleright \lambda s. \langle s, \text{self } s \rangle)$$

expand $\triangleright \dots$

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$$(\lambda s. \langle \mathbf{a}::s, \mathbf{a} \rangle) \triangleleft \lambda s. \langle s, \mathbf{hug}(se1s) \rangle$$

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$$\lambda s. \langle \mathbf{a}::s, \mathbf{hug}(\mathbf{sel}(\mathbf{a}::s))\mathbf{a} \rangle$$

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- expressions that introduce discourse referents or engage in anaphora engage with the environment
- values are trivially stateful, by passing the environment on, untouched

How might one do this?

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Answer: one may use *monad transformers* (the strategy adopted by Shan (2002), and then, by Charlow (2014)).

\mathcal{C} and \mathcal{S} are associated with corresponding monad transformers, \mathcal{C}_T and \mathcal{S}_T .

Definition (\mathcal{M}_T)

$$\mathcal{M}_T : (\mathcal{T} \rightarrow \mathcal{T}) \rightarrow \mathcal{T} \rightarrow \mathcal{T}$$

$$(\cdot)^\eta : a \rightarrow \mathcal{M}_T(\mathcal{M}_0)(b)$$

$$(\gg) : \mathcal{M}_T(\mathcal{M}_0)(a) \rightarrow (a \rightarrow \mathcal{M}_T(\mathcal{M}_0)(b)) \rightarrow \mathcal{M}_T(\mathcal{M}_0)(b)$$

given one of \mathcal{C} or \mathcal{S} as the *underlying monad*, we may apply one of \mathcal{S}_T or \mathcal{C}_T to it...

Definition (\mathcal{C}_T)

$$\mathcal{C}_T(\mathcal{M}_0)(a) : (a \rightarrow \mathcal{M}_0(o)) \rightarrow \mathcal{M}_0(o)$$

$$(\cdot)^\eta : a \rightarrow (a \rightarrow \mathcal{M}_0(o)) \rightarrow \mathcal{M}_0(o)$$

$$v^\eta = \lambda c. cv$$

$$(\gg) : ((a \rightarrow \mathcal{M}_0(o)) \rightarrow \mathcal{M}_0(o))$$

$$\rightarrow (a \rightarrow (b \rightarrow \mathcal{M}_0(o)) \rightarrow \mathcal{M}_0(o))$$

$$\rightarrow (b \rightarrow \mathcal{M}_0(o)) \rightarrow \mathcal{M}_0(o)$$

$$m \gg k = \lambda c. m(\lambda v. kv c)$$

Definition (\mathcal{S}_T)

$$\mathcal{S}_T(\mathcal{M}_0)(a) : s \rightarrow \mathcal{M}_0((s, a))$$

$$(\cdot)^\eta : a \rightarrow s \rightarrow \mathcal{M}_0((s, a))$$

$$v^\eta = \lambda s. \langle s, v \rangle^\eta$$

$$(\gg) : (s \rightarrow \mathcal{M}_0((s, a)))$$

$$\rightarrow (a \rightarrow (s \rightarrow \mathcal{M}_0((s, b))))$$

$$\rightarrow s \rightarrow \mathcal{M}_0((s, b))$$

$$m \gg k = \lambda s. ms \gg \lambda p. \text{let } \langle s', v \rangle = p \text{ in } kvs'$$

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 - ▶ but a generic meaning for *every* cannot be written... we are required to know what \mathcal{M}_0 is!
 - ▶ even then, the meaning the quantifier will be somewhat stipulative, e.g., to account for the data above (though, it can be made to follow from a small set of primitives, as in Charlow (2014))

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Might we salvage our individual analyses in some other way? In doing so, might we account for data like (1)?

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- 2 Algebraic effects and handlers**
- 3 Making it Montagovian
- 4 Quantification and dynamism

Algebraic effects and handlers provide a means of writing extensible code, recently especially popular in functional programming.¹

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- sticking with a traditional analysis of quantifiers, i.e., whereon they denote sets of sets

Algebraic signatures

An algebraic signature is a set E of operations, each one associated with a *parameter* p and an *arity* a (both types), along with a special operation η ('return').

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η always has the following type signature:

$$\eta : v \rightarrow \mathcal{F}_E(v)$$

In addition to the signature, an algebra determines a set of equations that must hold among its elements, of the form

$$\text{op}_i(p_i; \dots) = \text{op}_j(p_j; \dots)$$

The State algebra (signature)

instead of a State monad, we will have a State algebra

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The State algebra (laws)

Reading the environment twice is no better than reading it once:

$$\text{get}_{\star \rightsquigarrow s}(\star; \lambda g. \text{get}_{\star \rightsquigarrow s}(\star; \lambda g'. k g g')) = \text{get}_{\star \rightsquigarrow s}(\star; \lambda g. k g g)$$

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Putting twice overwrites:

$$\text{put}_{s \rightsquigarrow \star}(g; \lambda \star. \text{put}_{s \rightsquigarrow \star}(g'; k)) = \text{put}_{s \rightsquigarrow \star}(g'; k)$$

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Some example elements of the Quantifier algebra...

- $\text{scope}_{(e \rightarrow t) \rightarrow t \rightsquigarrow e}(\text{everydog}; \lambda y. \eta(\text{sleepy})) : \mathcal{F}_{\{\text{scope}_{(e \rightarrow t) \rightarrow t \rightsquigarrow e}\}}(t)$
- $\text{scope}_{(e \rightarrow t) \rightarrow t \rightsquigarrow e}(\text{everydog}; \lambda y. \text{scope}_{(e \rightarrow t) \rightarrow t \rightsquigarrow e}(\text{everycat}; \lambda z. \eta(\text{chasezy}))) : \mathcal{F}_{\{\text{scope}_{(e \rightarrow t) \rightarrow t \rightsquigarrow e}\}}(t)$

Quantifying in:

$$\text{scope}_{(e \rightarrow t) \rightarrow t \rightsquigarrow e}(q; \lambda x. \eta(kx)) = \eta(qk)$$

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Commuting $\text{scope}_{(e \rightarrow t) \rightarrow e \rightsquigarrow e}$ past $\text{get}_{\star \rightsquigarrow s}$ and $\text{put}_{s \rightsquigarrow \star}$:

$$\begin{aligned} & \text{scope}_{(e \rightarrow t) \rightarrow e \rightsquigarrow e}(q; \lambda x. \text{get}_{\star \rightsquigarrow s}(\star; \lambda s. \text{put}_{s \rightsquigarrow \star}(s'; \lambda \star. kxss'))) \\ &= \text{get}_{\star \rightsquigarrow s}(\star; \lambda s. \text{put}_{s \rightsquigarrow \star}(s; \lambda \star. \text{scope}_{(e \rightarrow t) \rightarrow e \rightsquigarrow e}(q; \lambda x. kxss'))) \end{aligned}$$

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How to do it

What we want is an encoding of the operations, as well as a way of *translating* λ -terms with lots of operations into ones with fewer operations in a way that respects the algebraic laws.

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In the combined State/Quantifier algebra, the normal form for any element is determined by the laws to be

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for some $f : s \rightarrow s$ and $g : s \rightarrow v$.

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Pairs of such functions f and g can be represented as $\lambda s. \langle f s, g s \rangle$... they are State monadic!

Encoding elements

To encode elements of an algebra, we define a family of functors

$\mathcal{F} : \mathcal{T}_{\rightsquigarrow}^* \rightarrow \mathcal{T} \rightarrow \mathcal{T}$, where

- $\mathcal{T}_{\rightsquigarrow}^*$ is the free monoid (i.e., of lists) over $\mathcal{T}_{\rightsquigarrow} = \{p \rightsquigarrow a \mid p, a \in \mathcal{T}\}$

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$$\mathcal{F}_\epsilon(v) = v$$

$$\mathcal{F}_{p \rightsquigarrow a, l}(v) = (p \rightarrow (a \rightarrow \mathcal{F}_l(v)) \rightarrow o) \rightarrow o$$

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Operations construct “pairs”; returning just returns...

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```
scope(e→t)→t↔e(everydog;  
  λx.get★↔s(★;  
    λs.puts↔★(x::s;  
      λ★.get★↔s(★; λs'.η(hug(sel s')x))))))
```

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$$\begin{aligned} & \text{scope}_{(e \rightarrow t) \rightarrow t \rightsquigarrow e}(\text{everydog}; \\ & \quad \lambda x. \text{get}_{\star \rightsquigarrow s}(\star; \\ & \quad \quad \lambda s. \text{put}_{s \rightsquigarrow \star}(x :: s; \\ & \quad \quad \quad \lambda \star. \text{get}_{\star \rightsquigarrow s}(\star; \lambda s'. \eta(\mathbf{hug}(\text{self } s') x)))))) \\ & = \lambda h. h(\text{everydog})(\lambda x, h'. h' \star (\lambda s. \dots)) \end{aligned}$$

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 \end{aligned}$$

This will be an expression of type

$$\begin{aligned}
 & \mathcal{F}_{(e \rightarrow t) \rightarrow t \rightsquigarrow e, \star \rightsquigarrow s, s \rightsquigarrow \star, \star \rightsquigarrow s} \\
 & = (((e \rightarrow t) \rightarrow t) \rightarrow (e \rightarrow ((\star \rightarrow (s \rightarrow \dots)) \rightarrow o') \rightarrow o')) \rightarrow o) \rightarrow o
 \end{aligned}$$

We have a way of encoding meanings involving quantifiers and anaphora.

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What we would like is to provide a *handler* that implements our reduction rules, i.e., those determined by the algebraic laws.

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We need a family of functions

$$\text{handleSentence}_l : \mathcal{F}_l(t) \rightarrow \mathcal{F}_{\star \rightarrow s, s, \rightarrow \star}(t)$$

where $l \in \{(e \rightarrow t) \rightarrow t \rightsquigarrow e, \star \rightsquigarrow s, s \rightsquigarrow \star\}^*$.

- 1 Side effects in linguistic semantics
- 2 Algebraic effects and handlers
- 3 Making it Montagovian
- 4 Quantification and dynamism

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This law destroys a quantifier's dynamic potential, rendering it externally static.

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This gives us a new and precise way of characterizing certain old semantic problems about quantification and dynamism:

- when combining algebras, where do any new laws come from? can they come for free?

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